Universal dynamical control of quantum mechanical decay: Modulation of the coupling to the continuum

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We derive and investigate an expression for the dynamically modified decay rate of a state coupled to an *arbitrary* continuum. This expression is universally valid for weak temporal perturbations. The resulting insights can serve as useful recipes for optimized control of decay and decoherence.

PACS numbers: 03.65.Ta, 03.65.Xp, 42.25.Kb, 42.50.Vk

The quantum Zeno effect (QZE), namely, the *inhibition of the decay* of an unstable state by its (sufficiently frequent) projective measurements, has been long considered to be a basic *universal* feature of quantum systems [1]. Our recent general analysis [2] has revealed the *inherent impossibility* of the QZE for a broad class of processes, including spontaneous emission in open space, as opposed to the ubiquitous occurrence of the anti-Zeno effect, i.e., *decay acceleration* by frequent projective measurements [3]. Although realistic schemes may well approximate such measurements [2,4], there is strong incentive for raising the question: Are projective measurements the most effective way of modifying the decay of an unstable state? This question is prompted by two recent, important, results: (a) A landmark experiment has demonstrated, for the first time, both the inhibition and the acceleration of quantum-mechanical decay by repeated on-off switching of the coupling between a nearly bound state and the continuum, using a system of cold atoms that are initially trapped in an optical-lattice potential [5]. (b) It has been theoretically shown that a series of 2π-pulses, acting between the decaying level and an auxiliary one, can either inhibit or accelerate the decay into certain model reservoirs [6]. In both cases, the repeated interruption of the "natural" evolution is imperative for decay modification. By contrast, dephasing, which is an essential ingredient of projective measurements [4], is completely absent in Ref. [6].

In this paper we purport to substantially expand the arsenal of decay control methods, as well as elucidate the modifications of decay to *any* reservoir (continuum) by dynamical perturbations, whether measurement-like (i.e., accompanied by dephasing) or fully coherent. We derive a universal form of the dynamically modified decay rate of an unstable state, valid for weak time-dependent perturbations. The results of Refs. [2,3,5,6] are recovered as limiting cases of this universal form. Our analysis can serve as a general recipe for optimized decay control, be it decay and decoherence suppression for quantum logic operations [7] or decay enhancement for the control of chaos or chemical reactions [8].

Consider the decay of a state $|e\rangle$ via its coupling to a system, described by the orthonormal basis $\{|j\rangle\}$, which forms either a discrete or a continuous spectrum (or a mixture thereof). To modify the decay, we allow for a *dynamical modulation* of the interaction. In its most general form, the total Hamiltonian is a sum of

$$\hat{H}_0 = \hbar\omega_a|e\rangle\langle e| + \hbar\sum_j \omega_j|j\rangle\langle j|, \tag{1}$$

with $\hbar\omega_a$ and $\hbar\omega_j$ being the energies of $|e\rangle$ and $|j\rangle$, respectively;

$$\hat{V}(t) = \sum_j V_{ej}(t)|e\rangle\langle j| + \text{h.c.}, \tag{2}$$

denoting the off-diagonal coupling of $|e\rangle$ with the other states, which is deliberately temporally modulated, its *static form* describing the natural decay process; and

$$H_1(t) = \hbar\delta_a(t)|e\rangle\langle e| + \hbar\sum_j \delta_j(t)|j\rangle\langle j|, \tag{3}$$

standing for the adiabatic (diagonal) time-dependent perturbations of the energies of the initial ($|e\rangle$) and final ($|j\rangle$) states, e.g., AC Stark shifts.

We write the wave function of the system, with $|e\rangle$ populated at $t = 0$, as

$$|\Psi(t)\rangle = \alpha(t)e^{-i\omega_a t - i\int_0^t \delta_a(t')dt'}|e\rangle + \sum_j \beta_j(t)e^{-i\omega_j t - i\int_0^t \delta_j(t')dt'}|j\rangle, \tag{4}$$

the initial condition being $|\Psi(0)\rangle = |e\rangle$. Henceforth we treat the generic case, wherein the level shifts and the temporal modulation of $\hat{V}(t)$ are *independent* of j, i.e., $\delta_j(t) \equiv \delta_f(t)$ and $V_{je}(t) \equiv \tilde{\epsilon}(t)\mu_{je}$, $\tilde{\epsilon}(t)$ being the modulation function (Fig. 1 – inset). Such factorized form of the modulation is commonly valid for weak or moderate time-dependent fields, which do not appreciably change the states of the continuum. One then obtains from the Schrödinger equation that the amplitude $\alpha(t)$ obeys the *exact* integro-differential equation [9]

$$\dot{\alpha} = -\int_0^t dt'\epsilon^*(t)\epsilon(t')\Phi(t-t')e^{i\omega_a(t-t')}\alpha(t'), \tag{5}$$

where $\Phi(t-t') = \hbar^{-2}\sum_j |\mu_{ej}|^2 e^{-i\omega_j(t-t')}$ and $\epsilon(t) = \tilde{\epsilon}(t)\exp[-i\int_0^t \delta_{af}(t')dt']$, with $\delta_{af}(t) = \delta_a(t) - \delta_f(t)$. The function $\epsilon(t)$ accounts for the modulation of *either diagonal or off-diagonal elements* of the unperturbed Hamiltonian.

The assumption that the coupling (2) is a weak perturbation of (1) implies that $\alpha(t)$ *varies sufficiently slowly* with respect to the kernel of Eq. (5), so that one can make the approximation $\alpha(t') \approx \alpha(t)$ on the right-hand side (rhs) of Eq. (5). Then one can solve Eq. (5) and represent the population $P(t) = |\alpha(t)|^2$ of the level $|e\rangle$ in the form

$$P(t) = \exp[-R(t)Q(t)], \tag{6}$$

where we have introduced the fluence $Q(t) = \int_0^t d\tau |\epsilon(\tau)|^2$, and obtained the decay rate in the *universal form*

$$R(t) = 2\pi \int_{-\infty}^{\infty} d\omega G(\omega + \omega_a) F_t(\omega). \tag{7}$$

Here $G(\omega) = \pi^{-1}\mathrm{Re}\int_0^\infty dt e^{i\omega t}\Phi(t) = \hbar^{-2}\sum_j |\mu_{ej}|^2 \delta(\omega - \omega_j)$ is the coupling spectrum, i.e., the density of states weighted by the strength of the coupling to the continuum or reservoir; $F_t(\omega) = |\epsilon_t(\omega)|^2/Q(t)$, with $\epsilon_t(\omega) = (2\pi)^{-1/2}\int_0^t \epsilon(t')e^{i\omega t'}dt'$, is the (normalized to unity) spectrum of the modulation function $\epsilon(t)$ in the "window" $(0,t)$. The result (6), (7) is *valid to all orders of* t, i.e., it keeps intact the *interferences* between the modulated decay channels.

We now consider some important consequences of the universal form (6), (7). The modulation spectrum $F_t(\omega)$ is roughly characterized by its width ν_t and the frequency shift $\Delta_t = \int d\omega \omega F_t(\omega)$. *A modulation may strongly modify the decay rate* (analogously to the QZE or AZE) whenever $\nu_t + |\Delta_t| \gtrsim \xi(\omega_a)$, where $\xi(\omega_a)$ *is the characteristic spectral interval over which the weighted density of states* $G(\omega)$ *changes* near ω_a. In particular, if the continuum $G(\omega)$ is a bell-shaped peak and ω_a is within its width Γ_R, then $\xi(\omega_a) = \Gamma_R$ (Fig. 1b). If ω_a is near the *edge of the continuum* (as for radiative decay in photonic crystals or vibrational decay in molecules and solids), then $\xi(\omega_a)$ is the distance between ω_a and the edge [2] (Fig. 1a). Only in the opposite limit, $\nu_t + |\Delta_t| \ll \xi(\omega_a)$, can one approximately set $F_t(\omega) \approx \delta(\omega)$ in Eq. (7), yielding $P(t) \approx \exp[-R_{\mathrm{GR}}Q(t)]$, where $R_{\mathrm{GR}} = 2\pi G(\omega_a)$ is the *extension of the Golden-Rule (GR) rate to the case of a time-dependent coupling.*

The modulation function $\epsilon(t)$ can be either random or regular (coherent) in time. Consider first the most general coherent *amplitude and phase* modulation (APM) of the quasiperiodic form, $\epsilon(t) = \sum_k \epsilon_k e^{-i\omega_k t}$. Here ω_k ($k = 0, \pm 1, \ldots$) are arbitrary discrete frequencies with the minimum spectral distance Ω. For a given function $\epsilon(t)$ one can obtain $-i\omega_k$ and ϵ_k as the poles and residues, respectively, of the Laplace transform $\hat{\epsilon}(s)$. If $\epsilon(t)$ is periodic with the period Ω, then $\omega_k = k\Omega$, and ϵ_k become the Fourier components of $\epsilon(t)$. For a general quasiperiodic $\epsilon(t)$, one obtains

$$Q(t) = \epsilon_c^2 t + \epsilon_c^2 \sum_{k \neq l} \lambda_k \lambda_l^* \frac{e^{i(\omega_l - \omega_k)t} - 1}{i(\omega_l - \omega_k)}, \tag{8}$$

where $\epsilon_c^2 = \sum_k |\epsilon_k|^2$ equals the average of $|\epsilon(t)|^2$ over a period of the order of $1/\Omega$, $\lambda_k = \epsilon_k/\epsilon_c$ and

$$|\epsilon_t(\omega)|^2 = \epsilon_c^2 t \sum_k |\lambda_k|^2 S(\eta_k t/2)$$
$$+\epsilon_c^2 \sum_{k \neq l} \lambda_k \lambda_l^* \frac{1 + e^{i(\omega_l - \omega_k)t} - e^{i\eta_k t} - e^{-i\eta_l t}}{2\pi \eta_k \eta_l}. \tag{9}$$

Here $\eta_k = \omega - \omega_k$, whereas $S(\eta_k t/2) = 2\sin^2(\eta_k t/2)/\pi t \eta_k^2$ is a sinc-function of η_k normalized to 1.

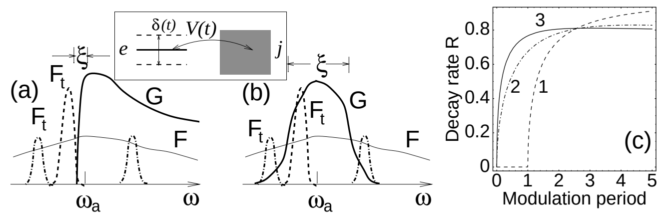

FIG. 1. Decay modification by modulation [Eqs. (6), (7)]. Inset: Schematic view of the temporal modulation of the shift of level e and its coupling to a continuum. (a) ω_a is near a band edge of $G(\omega) = C\omega^{1/2}(\omega + \Gamma)^{-1}\theta(\omega)$, where $\theta(\omega)$ is the unit step function; then [Eq. (12)] a small phase shift (dashed peak) is more effective in reducing the decay rate R than large phase shifts $\phi \simeq \pi$ (dash-dotted peaks) or frequent measurements/random $\epsilon(t)$ (thin curve). (b) ω_a is near a symmetric peak of $G(\omega)$; then large phase shifts (dash-dotted) are more effective than small phase shifts (dashed) and frequent measurements/random $\epsilon(t)$ (thin). (c) Decay rate R (in units of R_{GR}) in case (a) with $\omega_a = 0.1\Gamma$: reduction by PM [Eq. (10)] (curve 1 – $\phi = 0.1$, curve 2 – $\phi = \pi$) and frequent impulsive measurements [2] (curve 3 – QZE) as a function of perturbation period τ (in units of Γ^{-1}). Curve 1 gives the strongest reduction of R at a given τ.

For $t \gg \Omega^{-1}$ the first term on the rhs of (9) is a sum of peaks, whose spacings are much greater than their width $2/t$. The fast oscillating second term is also peaked at $\omega = \omega_k$, but we then find that the ratio of the first to the second terms, and that of their counterparts in (8), is $\sim (\Omega t)^{-1} \ll 1$. In the long-time limit, we then neglect these fast oscillating terms and obtain from Eqs. (6)-(9) that $P(t) = \exp[-R(t)\epsilon_c^2 t]$, where $R(t)$ in Eq. (7) now involves $F_t(\omega) \approx \sum_k |\lambda_k|^2 S(\eta_k t/2)$. For even longer times, exceeding the effective correlation time $t_c \equiv \max_k\{1/\xi(\omega_a + \omega_k)\}$, the functions $S(\eta_k t/2)$ become narrower than the respective characteristic widths of $G(\omega)$ around $\omega_a + \omega_k$, and one can set $S(\eta_k t/2) \approx \delta(\eta_k)$. Then Eq. (7) is reduced to

$$R = 2\pi \sum_k |\lambda_k|^2 G(\omega_a + \omega_k). \tag{10}$$

Hence, the long-time limit of the general decay rate (7) under the APM is a sum of the GR rates, corresponding to the resonant frequencies shifted by ω_k, with the weights $|\lambda_k|^2$. This formula provides a *simple general recipe* for manipulating the decay rate by APM. It holds if $Rt_c \ll 1$. The following limits of (10) will be now analyzed.

(i) *Monochromatic perturbation*: Let $|\lambda_k|^2 = \delta_{0k}$, where δ_{0k} is the Kronecker symbol. Then

$$R = 2\pi G(\omega_a + \Delta). \tag{11}$$

Here $\Delta \equiv \omega_0$ typically results from AC Stark shifts, $\Delta = \delta_{af}$. In principle, these shifts may drastically enhance or suppress R relative to $R_{\rm GR}$. Equation (11) provides the *maximal variation* of R achievable with an external perturbation, since it is not affected by any averaging (smoothing) of $G(\omega)$ due to the width ν of $F(\omega)$: As $|\Delta|$ grows, this R can decrease much faster than $1/|\Delta|$ (compare with the Zeno scaling $R \sim 1/\nu$ [2]) and even *vanish*, if $\omega_a + \Delta$ is beyond the cutoff frequency of the coupling, where $G(\omega) = 0$ (Fig. 1a,c). Likewise, the increase of R due to a shift can be much greater than that achievable with the AZE [2]. In practice, however, AC Stark shifts are usually small for (CW) monochromatic perturbations, whence other methods should often be used, requiring pulsed perturbations.

(ii) *Impulsive phase modulation (PM)*: Let the phase of the coupling amplitude jump by an amount ϕ at times $\tau, 2\tau, \ldots$. Such modulation can be achieved by a train of identical, equidistant, narrow pulses of nonresonant radiation, which produce pulsed AC Stark shifts $\delta_{af}(t)$ in (3). Now $\epsilon(t) = e^{i[t/\tau]\phi}$, where $[\ldots]$ is the integer part. One then obtains that $Q(t) = t$. According to Eq. (6), the decay at $t = n\tau$ has then the form $P(n\tau) = \exp[-R(n\tau)n\tau]$, where $R(n\tau)$ is defined by Eq. (7) with

$$F_{n\tau}(\omega) = \frac{2\sin^2(\omega\tau/2)\sin^2[n(\phi+\omega\tau)/2]}{\pi n\tau\omega^2\sin^2[(\phi+\omega\tau)/2]}. \tag{12}$$

We note that the case $\phi = \pi$ is equivalent to that proposed in [6]. For sufficiently long times one can use Eq. (10). Now $\epsilon_c = 1$, whereas the poles and residues of $\hat{\epsilon}(s) = (1-e^{-s\tau})/[s(1-e^{i\phi-s\tau})]$ yield $\omega_k = 2k\pi/\tau - \phi/\tau$ and $|\lambda_k|^2 = 4\sin^2(\phi/2)/(2k\pi-\phi)^2$. For *small phase shifts*, $\phi \ll 1$, the $k = 0$ peak dominates, $|\lambda_0|^2 \approx 1-\phi^2/12$, whereas $|\lambda_k|^2 \approx \phi^2/4\pi^2k^2$ for $k \neq 0$. In this case one can retain only the $k = 0$ term in Eq. (10) [unless $G(\omega)$ is very fast changing]. Then Eq. (10) reduces to (11), i.e., the modulation acts as a constant frequency shift $\Delta = -\phi/\tau$. With the increase of $|\phi|$, the difference between the $k = 0$ and $k = 1$ peak heights diminishes, *vanishing* for $\phi = \pm\pi$. Then $|\lambda_0|^2 = |\lambda_1|^2 = 4/\pi^2$, i.e., $F_t(\omega)$ for $\phi = \pm\pi$ contains *two identical peaks symmetrically shifted in opposite directions* (Fig. 1a,b) [the other peaks $|\lambda_k|^2$ decrease with k as $(2k-1)^{-2}$, totaling 0.19].

The above features allow one to adjust the modulation parameters for a given scenario to obtain an optimal decrease or increase of R. Generally, the PM scheme with a small ϕ is preferable, since it yields a spectral shift in the required direction (positive or negative). The adverse effect of $k \neq 0$ peaks in $F_t(\omega)$ then scales as ϕ^2 and hence can be significantly reduced by decreasing $|\phi|$ (Fig. 1a,c). On the other hand, if ω_a is near a symmetric peak of $G(\omega)$, R is reduced more effectively for $\phi = \pi$ (or, at least, $\phi \sim 1$), since then the main $F_t(\omega)$ peaks at ω_0 and ω_1 shift stronger with τ^{-1} than the peak at $\omega_0 = -\phi/\tau$ for $\phi \ll 1$ (Fig. 1b).

(iii) *Amplitude modulation (AM)* of the coupling arises, e.g., for radiative-decay modulation due to atomic motion through a high-Q cavity or a photonic crystal [10] or for atomic tunneling in optical lattices with time-varying lattice acceleration [5,11]. Let the coupling be turned on and off periodically, for the time τ_1 and $\tau_0 - \tau_1$, respectively, i.e., $\epsilon = 1$ for $n\tau_0 < t < n\tau_0 + \tau_1$ and $\epsilon = 0$ for $n\tau_0 + \tau_1 < t < (n+1)\tau_0$ $(n = 0, 1, \ldots)$. Now $Q(t)$ is the total time during which the coupling is switched on. The decay at $t = n\tau_0$ is described by $P(n\tau_0) = \exp[-R(n\tau_0)n\tau_1]$, where $R(n\tau_0)$ in Eq. (7) has

$$F_{n\tau_0}(\omega) = \frac{2\sin^2(\omega\tau_1/2)\sin^2(n\omega\tau_0/2)}{\pi n\tau_1\omega^2\sin^2(\omega\tau_0/2)}. \tag{13}$$

The parameters in Eq. (10) are now found to be $\epsilon_c^2 = \tau_1/\tau_0$, $\omega_k = 2k\pi/\tau_0$, $|\lambda_0|^2 = \tau_1/\tau_0$, $|\lambda_k|^2 = (\tau_1/\tau_0)\mathrm{sinc}^2(k\pi\tau_1/\tau_0)$ $(k \neq 0)$.

The results above imply that whenever τ_0 is so large that $\tau_1 \ll \tau_0$ and $G(\omega)$ does not change significantly over the spectral intervals $(2\pi k/\tau_0, 2\pi(k+1)/\tau_0)$ (i.e., τ_0 is *greater than the correlation time* of the continuum), one can approximate the sum (10) by the integral (7) with $F_t(\omega) \approx F(\omega) = (\tau_1/2\pi)\mathrm{sinc}^2(\omega\tau_1/2)$, characterized by the spectral broadening $\sim 1/\tau_1$. Then Eq. (7) for R *coincides* with that obtained when ideal projective measurements are performed at intervals τ_1 [2]. Thus the AM scheme *can emulate measurement-induced (dephasing) effects* on quantum dynamics. This indeed has been observed [5] for atom tunneling in optical lattices whose tilt (acceleration) was periodically modulated as above. For its analysis we use the approximate expression for $\Phi(t)$ obtained in [11], which yields the reservoir spectrum $G(\omega + \omega_a)$ (Fig. 2 – inset), with one maximum at $\omega \sim \omega_g$, $\hbar\omega_g$ being the lattice band gap. The calculated $F_t(\omega)$ effectively amounts to spectral broadening. The decay probability $P(t)$, calculated in Fig. 2 (curves 1-4) for parameters similar to [5], *completely coincides* with that obtained for ideal impulsive measurements at intervals τ_1 [2] and demonstrates either the QZE (curve 2) or the AZE (curve 3) behavior.

Let us, however, decrease τ_1, keeping $\tau_1/\tau_0 = $ const. Then $R_{\tau_1\to 0} = (\tau_1/\tau_0)R_{\rm GR}$, i.e., $R_{\tau_1\to 0}$ is nonvanishing, though somewhat reduced in comparison with $R_{\rm GR}$. This is *a peculiar non-QZE* behavior, hitherto unobserved (curve 5 in Fig. 2).

Remarkably, the universal Eq. (7) is valid also when $\epsilon(t)$ is a *stationary random process*, characterized by one time scale, the correlation time ν^{-1}. Then one can use a master equation to show that, for $t \gg \nu^{-1}$, we have $P(t) \approx e^{-Rt}$, where the decay rate (provided that $R \ll \nu$) still has the general form (7), but with

$$F_t(\omega) \to F(\omega) = \pi^{-1}\epsilon_c^{-2}\mathrm{Re}\int_0^\infty \overline{\epsilon^*(t)\epsilon(0)}e^{i\omega t}dt, \quad (14)$$

$F(\omega)$ being the normalized spectrum of the random process and $\epsilon_c^2 = \overline{|\epsilon(t)|^2}$, where the overbar denotes ensemble averaging. Expression (7) with the substitution (14) is *completely analogous* to the universal formula describing *measurement effects* on quantum evolution in [2]: It extends the results of Ref. [2] to the effects of *any* random $\epsilon(t)$, whether governed by AM, PM or APM.

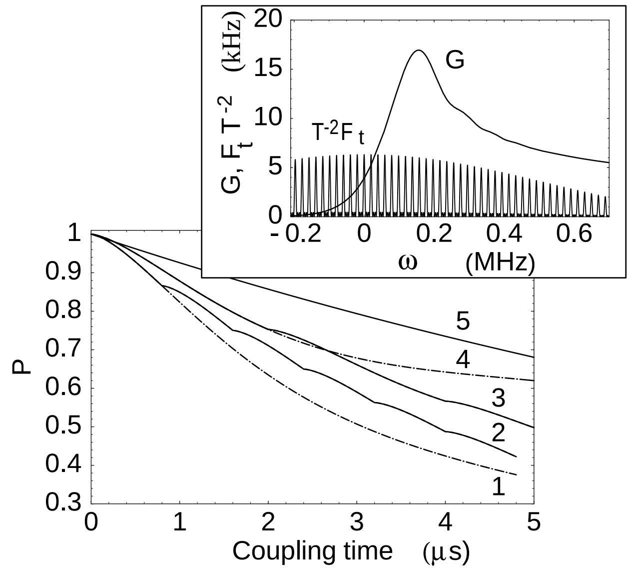

FIG. 2. Tunneling of sodium atoms in optical lattices perturbed by AM scheme [Eq. (13)]: the decay probability $P(t)$ as a function of the total coupling time. Curves 1, 4 – decay without modulation. Curve 2 – QZE (decay slowdown compared to curve 1) for $\tau_1 = 0.8$ μs, $\tau_0 = 50.8$ μs. Curve 3 – AZE (decay speedup compared to curve 4) for $\tau_1 = 2$ μs, $\tau_0 = 52$ μs. Curve 5 – *non-QZE* decay slowdown (compared to curve 1) for $\tau_1 \to 0$ with $\tau_1/\tau_0 = 0.5$. Inset: The coupling spectrum $G(\omega+\omega_a)$ and the scaled modulation function $T^{-2}F_{4\tau_0}(\omega)$ for the conditions of curve 2. Here $T = \omega_g d/(\pi a)$, where $a = 15$ km/s^2 is the lattice acceleration and $d = 295$ nm is the lattice period. $\omega_g = 91$ kHz, $\omega_g T = 2.05$ (for curves 1, 2, 5); $\omega_g = 116$ kHz, $\omega_g T = 3.32$ (for curves 3, 4).

Typically, $F(\omega)$ in Eq. (14) a bell-shaped function characterized by its width ν and shift $\Delta = \int d\omega\omega F(\omega)$. Projective measurements at an effective rate ν, whether impulsive or continuous, usually result in broadening (to a width ν) of $F(\omega)$, without a shift of its maximum, $\Delta \approx 0$ [2,4]. This feature was shown [2] to be responsible for either standard QZE scaling, $R \sim 1/\nu$, or the AZE scaling. In contrast, consider a random Stark shift $\delta_{af}(t) = \chi I(t)$ caused by a chaotic field with the Lorentzian spectrum of the HWHM width ν_0, where $I(t)$ is the intensity and χ is the effective polarizability. If the field is weak and broadband, $|\chi|\overline{I} \ll \nu_0$, the dephasing function $F(\omega)$ is a Lorentzian with a substantial shift $\Delta = \chi\overline{I}$, which is much larger than the HWHM width $\nu = \chi^2\overline{I}^2/\nu_0$. If, in addition, ν satisfies the generalized GR criterion $\nu \ll \xi(\omega_a + \Delta)$, then Eq. (14) is reduced to $F(\omega) \approx \delta(\omega - \Delta)$, i.e., the random dephasing effectively acts on R as a monochromatic perturbation [Eq. (11)]. This dependence of R on Δ is *much stronger* than the QZE (or AZE) dependence on ν.

We have presented here a general theory of dynamically controlled decay. Its unified form (6), (7) encompasses, as special cases, all the modulation schemes of current interest, satisfying the factorization condition [cf. Eq. (5)] [5,6,12,13]. Whereas its limits (13), (14) may imitate measurement effects (the QZE and AZE), the modulation parameters allow us to "engineer" more effectively the decay into a given reservoir. The coherence of APM makes it appropriate for decoherence suppression in quantum information applications, which require reversible transformations of quantum superposed states.

This work has been supported by the US-Israel BSF (G. K.), the EU (ATESIT Network) and the Ministry of Absorption (A. K.).

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